

The Earth's Climate Sensitivity and Thermal Inertia

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Abstract

The Earth's *equilibrium climate sensitivity* has received much attention because of its relevance and importance for global warming policymaking. This paper focuses on the Earth's *thermal inertia time scale* which has received relatively little attention. The difference between the observed *transient climate sensitivity* and the equilibrium climate sensitivity is shown to be proportional to the thermal inertia time scale, and the numerical value of the proportionality factor is determined using recent observational data. Many useful policymaking insights can be extracted from the resulting empirical quantitative relation.

1 Introduction

For millennia prior to the industrial revolution, the global surface temperature of the Earth, $T_s(t)$, fluctuated about a constant average value $T_{s,o}$ ($\approx 288^\circ K$), while the atmospheric carbon dioxide concentration $X(t)$ fluctuated about a constant average value X_o (≈ 285 ppmv). Subsequent to 1850, the value of $X(t)$ has increased significantly above X_o , and the observed average global surface temperature $T_s(t)$ has also increased. Since carbon dioxide is a major long resident time greenhouse gas, it is reasonable to associate the observed increase of $T_s(t)$ to the increase of $X(t)$ and other atmospheric greenhouse gases. A useful quantitative measure of atmospheric carbon dioxide driven global warming is the estimated amount of average warming ΔT_s (above the pre-industrial revolution value $T_{s,o}$) when X/X_o is held at some constant value until the Earth system reaches *steady state equilibrium*. This *equilibrium global surface temperature rise* is denoted by $\Delta T_s^{eq}(X/X_o)$. The *Intergovernmental Panel on Climate Change* (IPCC) (Solomon *et al.*, 2007; Houghton *et al.*, 2001, 1995, 1990) recommends the following relationship for $\Delta T_s^{eq}(X/X_o)$:

$$\Delta T_s^{eq}(X/X_o) \equiv \frac{\lambda_{2X_o}^{eq}}{\ln 2} \ln \left(\frac{X}{X_o} \right). \quad (1)$$

While $\Delta T_s^{eq}(X/X_o)$ depends logarithmically on X/X_o , it is merely proportional to $X/X_o - 1$ for X/X_o close to unity.

The constant parameter $\lambda_{2X_o}^{eq}$ in Eq.(1) is called the *equilibrium climate sensitivity* of the Earth for carbon dioxide. It is the average global surface temperature rise that results when X is held constant at $2X_o$ until steady state is reached. The numerical value of $\lambda_{2X_o}^{eq}$ is crucial to global warming policymaking. The IPCC suggested $2^\circ C \leq \lambda_{2X_o}^{eq} \leq 4.5^\circ C$ as its “likely range,” and $\lambda_{2X_o}^{eq} \approx 3^\circ C$ as its “best estimate” value. These suggestions were based largely on assessments of data generated by the best available computer climate models (Houghton *et al.* 1990, Box 10.2; also Andronova and Schlesinger, 2001). After acknowledging that the numerical value of $\lambda_{2X_o}^{eq}$ “... is a key uncertainty ...”, the IPCC nevertheless said “... values substantially higher than $4.5^\circ C$ cannot be excluded, but agreement of models with observations is not as good for those values.” (Solomon *et al.*, 2007, §2.3, Chapter 10, p.798. See also Houghton *et al.*, 1990, §8.6, §9.6, SPM). This statement implies that the *probability distribution function* (PDF) of $\lambda_{2X_o}^{eq}$ might have a “fat-tail,” an issue of concern in the policy community (Weitzman, 2009).

2 Historical observational data

Fig.1 plots observational average global surface temperature ΔT_s data versus $\ln(X/X_o)$ between 1850-2012 (Brohan *et al.*, 2006; Enting *et al.* 1994; Morice *et al.* 2012). The $X(t)$ data is a relatively smoothly rising function of time (See Fig. 1 in Hansen *et al.*, 2005). The data for $\ln(X/X_o) < 0.13$ were taken before 1970. The scatter of the pre-1970 data is significantly larger than that of the post-1970 data (see also Fig. 4A in Hansen *et al.*, 2010).

Inspection of Fig.1 suggests the following formula for the post-1970 observed data:

$$\overline{\Delta T_s}^{obs}(t) = \frac{\lambda_{2X_o}^{obs}}{\ln 2} \ln \left(\frac{X(t)}{X_o} \right), \quad (2)$$

where the overline notation denotes some “best-fit” representation of the data over some “sufficiently long” time interval, and $\lambda_{2X_o}^{obs}$, the transient climate sensitivity (Held, Winton, Takahashi *et al.*, 2010), is a constant to be empirically determined by curve-fitting. From a least squares fit to the post-1970 data, one obtains $\lambda_{2X_o}^{obs} \approx 2.3^\circ C$ ($R=0.88$). In Fig.1, the bold line is this best-fit $\overline{\Delta T_s}^{obs}(t)$ line,¹ while the dashed line is $\Delta T_s^{eq}(t)$ plotted using Eq.(1) with $\lambda_{2X_o}^{eq} = 4.5^\circ C$ —which is at the upper limit of the IPCC likely range. The divergence between these two lines is clearly shown with observed values following the $2.3^\circ C$ line. A good correlation is necessary for the cause-and-effect connection between the observed warming and atmospheric carbon dioxide, but of course it is not sufficient to “prove” the connection.

The value of $\lambda_{2X_o}^{obs}$ is the amount of average temperature rise *at the moment* when a *steadily rising* $X(t)$ reaches twice the pre-industrial revolution level. Obviously, the value of $\lambda_{2X_o}^{obs}$ depends on the rate of increase in $X(t)$. When $X(t)$ increases at 1% per year, the value of $\lambda_{2X_o}^{obs}$ is called the *transient climate response* (TCR). (See Fig. 10.25 of Houghton *et al.*, 1990). The post-1970 $X(t)$ observational data is indeed steadily rising at about 0.5% per year and if this rate is small enough, then the observed value of $\lambda_{2X_o}^{obs}$ should be a good approximation to $\lambda_{2X_o}^{eq}$.

¹Note that the bold line in Fig. 1 misses the origin by a small amount because it is the best-fit line only for the post-1970 data points. If all the pre-1970 ΔT_s data in Fig. 1 were also included, the resulting best-fit line would pass through the origin with $\lambda_{2X_o}^{obs} \approx 1.9^\circ C$ (correlation coefficient $R = 0.86$).

2.1 Rising rate of $\Delta T_s(t)$

Fig. 2 is a plot of the post-1970 observational $\Delta T_s(t)$ data versus time (Brohan *et al.*, 2006; see also Fig. 2 in Hansen *et al.*, 2005). It is seen that this data exhibits considerable scatter that can be attributed to natural random events such as El Niños, solar irradiance variations, albedo changes caused by major volcanic eruptions, etc. A more meaningful value of $d\Delta T_s/dt$ would be the slope of some *appropriately chosen* best-fit $\overline{\Delta T_s}^{obs}(t)$ versus time line.

A simple exponential is chosen to represent the post-1970 observational data, that is, $\overline{\Delta T_s}^{obs}(t)$ is assumed to be described by the following differential equation:

$$\frac{d\overline{\Delta T_s}^{obs}}{dt} = \frac{\overline{\Delta T_s}^{obs}}{\tau_T^{obs}}, \quad (3)$$

where the constant τ_T^{obs} is to be empirically determined by curve-fitting. Using least-squares, the value τ_T^{obs} of the post-1970 $\Delta T_s(t)$ data is found to be approximately 32 years ($R = 0.90$). The resulting best-fit $\overline{\Delta T_s}^{obs}(t)$ is the bold solid line in Fig. 2.

Of course, other time dependencies could have been chosen instead of Eq.(3) to curve-fit the post-1970 $\Delta T_s(t)$ data. The exponential time dependence was chosen because it has the correct *qualitative* behavior.

3 Energy balance of the Earth

The difference between the transient and the equilibrium ΔT_s responses is attributed to the finite thermal inertia of the Earth system (Hansen *et al.*, 2005). The Earth receives radiant energy from the Sun, and emits radiant energy back into space. The mismatch of these two fluxes changes the thermal energy content of the Earth system. Using a control volume enclosing the Earth (with its boundary surface *above* the atmosphere), the Earth's energy balance equation is:

$$\frac{dE}{dt} = F_{in}(t) - F_{out}(X, T_s, \dots), \quad (4)$$

where E is the stored thermal energy content per unit area of the Earth (Joules per unit area), and $F_{in}(t)$ and $F_{out}(X, T_s, \dots)$ are, respectively, the incoming radiant energy flux from the Sun and the outgoing radiant energy flux away from the Earth—both fluxes (Joules per year

per unit area) evaluated *at the top of the atmosphere*. The left hand side represents the thermal inertia of the Earth system.

3.1 Formulation and assumptions

For millennia prior to the industrial revolution, the Earth's $T_s(t)$ fluctuated about a constant steady state value $T_{s,o}$ of circa $288^\circ K$. Current warming of the Earth is assumed to be caused mainly by the greenhouse effects of increasing atmospheric carbon dioxide $X(t)$. The amount of equilibrium warming, Eq.(1), contains a single time-independent parameter $\lambda_{2X_o}^{eq}$, and it is desired to extract as much information as possible about $\lambda_{2X_o}^{eq}$ from the post 1970 observational data without getting too deeply involved with the detailed physics of the atmosphere and the oceans (Bierbaum *et al.*, 2003, Gregory *et al.*, 2002).

To this end, two simplifying assumptions are adopted to formulate the problem:

1. $E(T_s)$ depends only on T_s . This is called the *single energy reservoir* assumption which can be justified when all faster thermal reservoirs have equilibrated with each other so that a single slower reservoir dominates the system. (See Held *et al.* (2010), Stouffer (2010), and Socolow and Lam (2007) for the details of the mathematics.)
2. $F_{out} = F_{out}(X, T_s)$, i.e. F_{out} depends only on X and T_s , and nothing else. This assumes that atmospheric carbon dioxide, which has a long atmospheric life time, is the dominant cause of the global warming problem. It is known that there are other greenhouses gases in the atmosphere (e.g. methane, which has a much shorter atmospheric residence time than carbon dioxide). Wigley, Jones and Raper (1997; see its Fig. 5) suggested that atmospheric aerosols can also play a major role in the energy balance equation, contributing negative radiative forcing by reflecting incoming sunlight. Eq.(1) does not account for such effects.

Atmospheric water vapor and other feedback effects will be dealt with in §6. The role played by the thermal inertia term in Eq.(4) will be explored.

3.2 Linearized response to perturbations

When $F_{in}(t)$ and $X(t)$ are perturbed from their pre-industrial revolution steady state values $F_{in,o}$ and X_o by $\delta F_{in}(t, \dots)$ and $\delta X(t)$, the linearized equation governing the response of the surface temperature ΔT_s , derived from Eq.(4), is:

$$\frac{dE}{dT_s} \frac{d\Delta T_s}{dt} = \delta F_{net}(t, X, \dots) - \left(\frac{\partial F_{out}}{\partial T_s} \right)_X \Delta T_s, \quad (5)$$

where dE/dT_s is an *effective specific heat* per unit area of the Earth, and $\delta F_{net}(t, X, \dots)$ is the net incremental amount of *radiative forcing* per unit area of the Earth:

$$\delta F_{net}(t, X, \dots) \equiv \delta F_{in}(t, \dots) - \left(\frac{\partial F_{out}}{\partial X} \right)_{T_s} \delta X(t). \quad (6)$$

The last term in Eq.(5) represents the incremental amount of energy being radiated away from a warmer Earth. On the right hand side of Eq.(6), the first term $\delta F_{in}(t, \dots)$ accounts for all the natural random perturbations such as variations of solar irradiance, albedo changes caused by volcanic eruptions, etc., while the second term accounts for the *direct* greenhouse effect due to the increase of atmospheric carbon dioxide.

Assuming $(\partial F_{out}/\partial T_s)_{X_o} \neq 0$ and dividing Eq.(5) through by it, one obtains (North, Cahalan and Coakley, 1981; Schwartz, 2007, 2008. See also Held, Winton, Takahashi *et al.* 2010):

$$\tau_* \frac{d\Delta T_s}{dt} = \delta T_f(t, X, \dots) - \Delta T_s, \quad (7)$$

where

$$\tau_* \equiv \frac{\frac{dE}{dT_s}}{\left(\frac{\partial F_{out}}{\partial T_s} \right)_X}, \quad (8a)$$

$$\delta T_f(t, X, \dots) \equiv \frac{\delta F_{net}(t, X, \dots)}{\left(\frac{\partial F_{out}}{\partial T_s} \right)_X}. \quad (8b)$$

Under the single energy reservoir assumption, τ_* is formally a constant. However, it has been shown that for a multi-reservoir system τ_* could be time-dependent (see Appendix B of Socolow and Lam, 2007).

Once $\tau_*(t)$ is somehow specified, Eq.(7) can be used to compute the $\Delta T_s(t)$ response to any *given* forcing function $\delta T_f(t, X(t), \dots)$ of

interest. Since Eq.(7) is linear, the response to any additive contribution to $\delta T_f(t, X(t), \dots)$ can be separately studied (e.g. methane, sulfur aerosols, etc.).

3.3 Time-averaging

The actual forcing function $\delta T_f(t, X, \dots)$ on the Earth system contains all the unavoidable natural random disturbances. Thus the observed $\Delta T_s(t)$ data must also contain random components. Formally, zero-mean randomness can always be removed in such data by taking a running time-average over some sufficiently long time interval. Applying running time-averaging to Eq.(7) and denoting all the time-averaged entities using the overline notation, gives:

$$\tau \frac{d\overline{\Delta T_s}}{dt} = \overline{\delta T_f(t, X, \dots)} - \overline{\Delta T_s}. \quad (9)$$

where a new time scale parameter τ is formally introduced to replace $\tau_*(t)$. The subtle distinction between τ and τ_* will be discussed in §5.

Eq.(9) governs the time-averaged response of the Earth's system $\overline{\Delta T_s}(t)$ as driven by a *completely general* time-averaged forcing function $\overline{\delta T_f(t, X, \dots)}$. The parameter τ is as yet unknown.

4 The use of $\overline{\Delta T_s}^{obs}(t)$

It is assumed that the time-averaged observational data, $\overline{\Delta T_s}^{obs}(t) \approx \overline{\Delta T_s}$, honors Eq.(9). Using Eq.(3) to eliminate $d\overline{\Delta T_s}^{obs}/dt$ from Eq.(9), gives:

$$\overline{\delta T_f(t, X, \dots)} \approx \left(1 + \frac{\tau}{\tau_T^{obs}}\right) \overline{\Delta T_s}^{obs}(t). \quad (10)$$

This $\overline{\delta T_f(t, X, \dots)}$ forcing function is *empirically consistent* with the actual post-1970 $\overline{\Delta T_s}^{obs}(t)$ observed data for any positive τ . The amount of unrealized global warming “in the pipeline” (Hansen *et al.*, 2005) is thus $(\tau/\tau_T^{obs})\overline{\Delta T_s}^{obs}(t)$.

Note that no detailed physics was required in the derivation of Eq.(10). The crucial enabling step was the successful curve-fitting of the post-1970 $\Delta T_s(t)$ observational data to a simple exponential.

Using Eq.(2) for $\overline{\Delta T_s}^{obs}(t)$, one obtains:

$$\overline{\delta T_f(t, X, \dots)} \approx \left(1 + \frac{\tau}{\tau_T^{obs}}\right) \frac{\lambda_{2X_o}^{obs}}{\ln 2} \ln\left(\frac{X}{X_o}\right). \quad (11)$$

Comparing Eq.(11) to Eq.(1), a simple formula for the equilibrium climate sensitivity $\lambda_{2X_o}^{eq}$ is obtained:

$$\lambda_{2X_o}^{eq} \approx \left(1 + \frac{\tau}{\tau_T^{obs}}\right) \lambda_{2X_o}^{obs}. \quad (12)$$

The right hand side of Eq.(12) contains the unknown τ parameter. The values of the other two parameters, $\lambda_{2X_o}^{obs} \approx 2.3^\circ C$ and $\tau_T^{obs} \approx 32$ years, have been determined from the post-1970 observational data using least-squares curve-fitting.

Interesting questions are: is the order of magnitude of τ bigger or smaller than 32 years? Can τ_* be determined empirically by examining the actual $\Delta T_s(t)$ data? Can the order of magnitude of τ be estimated from such empirical values of τ_* ?

5 Observational estimates of τ

The Earth system is expected to exhibit a number of different thermal response time scales depending on the mode of excitation. The governing equation for $\Delta T_s(t)$ is Eq.(7), and the role played by τ_* on the $\Delta T_s(t)$ response to either a periodic or a Dirac-delta forcing function is well known. Since Earth's orbit around the sun has a small eccentricity, the solar radiation flux arriving on the Earth has a significant annual periodic variation ($\delta F_{net}(t)/F_{in,o} \approx 0.03 \sin(2\pi t)$). Douglass, Blackman and Knox (2004) analyzed the relevant observational data and found that the value of τ_* inferred from the phase shift of the data is less than one year (i.e. $2\pi\tau_* = O(1)$), while that inferred from the amplitude data is significantly larger than one year (i.e. $2\pi\tau_* \gg 1$). They favored the small τ_* inference from the phase shift data, and explained the discrepancy with the amplitude inference by a large *negative* feedback factor. The 1991 Pinatubo eruption is known to have affected the world's weather for several years. The commonly accepted explanation is that volcanic aerosols released into the atmosphere reduced incoming solar radiation by reflection (i.e. negative radiative forcing), and the residence time of such aerosols in the atmosphere is several years. Thus the 1991 Pinatubo eruption can be approximated by a Dirac-delta forcing function. Douglass and Knox (2005, 2006) analyzed the available observational data, and concluded that the inferred value of τ_* was less than one year.

The derivations of Eq.(7) and Eq.(9) both adopt the *single energy reservoir assumption*. The real Earth system obviously has more than

one thermal energy reservoir, and all the participating reservoirs are expected play a role. Thus the empirically determined τ_* 's discussed above were the time scales of the fast reservoirs which responded to the high frequency forcing. The value of τ of interest in this paper is the time scale of the slower reservoir which responded to the radiative forcing in the post-1970 time period. The bottom line is that τ and τ_* are not the same numerically. The values of the τ_* 's can only provide a lower limit to the order of magnitude of τ .

6 The physics of τ

The difficulties of modeling and calculating τ using first principles can be appreciated by assuming $\tau \approx \tau_*$ and attempting it with Eq.(8a).

The numerator $(dE/dT_s)_o$ on the right hand side of Eq.(8a) is the effective specific heat per unit surface area of the Earth. Assuming the oceans to be the Earth's sole energy reservoir, one can represent $(dE/dT_s)_o$ by the product of the per unit volume specific heat of water ($4.2 \times 10^6 \text{ Joule}/\text{m}^3\text{-}^\circ\text{C}$) and an *effective energy storage depth* H of the oceans:

$$\begin{aligned} \left(\frac{dE}{dT_s}\right)_o &= H \times 4.2 \times 10^6 \quad (\text{Joule}/\text{m}^2\text{-}^\circ\text{C}) \\ &= 0.13H \quad (\text{W-year}/\text{m}^2\text{-}^\circ\text{C}), \end{aligned} \quad (13)$$

where H is in meters. *All the difficult physics of mass and energy transport in the oceans is contained in H .*

The denominator $(\partial F_{out}/\partial T_s)_{X_o}$ on the right hand side of Eq.(8a) is determined by the detailed physics of radiative energy transport in the atmosphere. The outgoing radiation flux $F_{out}(T_s, X)$ at the top of the atmosphere is the sum of (i) the black body radiation emitted at the Earth's surface and (ii) the thermal radiation emission of the atmosphere itself, both *duly attenuated* by atmospheric absorption as they emerge from the atmosphere. One may formally represent $F_{out}(T_s, X)$ by:

$$F_{out}(T_s, X) = \hat{\epsilon}(T_s, X, \dots)\sigma T_s^4 \quad (\text{W}/\text{m}^2), \quad (14)$$

where $\hat{\epsilon}(T_s, X, \dots)$ is the *effective grey body emissivity* of the Earth at the top of the atmosphere looking downward, and σ is the Stefan-Boltzmann constant. *All the difficult physics inside the atmosphere (i.e. the T_s dependence of atmospheric water vapor, clouds, glaciers,*

conditions of the troposphere, etc.) is contained in the $\hat{\epsilon}(T_s, X, \dots)$ factor.

Taking the partial derivative of Eq.(14) with respect to T_s , assuming $\hat{\epsilon}(T_s, X)$, and rearranging, one obtains:

$$\left(\frac{\partial F_{out}}{\partial T_s}\right)_{X_o} = (4 - \phi_{fb}) \frac{F_{out,o}}{T_{s,o}}, \quad (15)$$

where ϕ_{fb} is:

$$\phi_{fb} \equiv - \left(\frac{\partial \ln \hat{\epsilon}}{\partial \ln T_s}\right)_{X_o}. \quad (16)$$

This dimensionless ϕ_{fb} represents the net ‘‘feedback factor’’ of the Earth’s atmosphere. The conventional wisdom is that ϕ_{fb} is positive, and that its value is dominated by water vapor feedback. Water vapor alone cannot be responsible for climate change because its atmospheric concentration is controlled by the atmospheric temperature distribution. Atmospheric Carbon dioxide, however, modifies this distribution via the greenhouse effect (Stevens and Bony, 2013). *First principle estimates of ϕ_{fb} have large uncertainties because the T_s dependence of $\hat{\epsilon}(T_s, X, \dots)$ is very difficult to model.* Note also that $(4 - \phi_{fb})$ is the ‘‘effective T_s exponent’’ of Eq.(14). Since Eq.(7) has been stable for millennia, $(4 - \phi_{fb})$ must be positive to be consistent with this historical fact.

The values of dE/dt (W/m^2) and $d\Delta T_s/dt$ ($^{\circ}C/year$) are related by $(dE/dT_s)_o$ which is represented by Eq.(13):

$$\frac{dE}{dt}(W/m^2) = 0.13H \times \frac{d\Delta T_s}{dt}(^{\circ}C/year). \quad (17)$$

Recently, Schwartz et al. (2010) carefully reviewed the literature on the available observational data of upper ocean heat content (Gregory et al., 2002; Schwartz, 2007; Willis et al., 2004; Gouretski and Koltermann, 2007; Wijffels et al., 2008), and recommended $\overline{dE/dt}^{obs} \approx 0.37 W/m^2$ —which is significantly smaller than the value ($0.60 W/m^2$) previously recommended by Hansen et al. (2005). Using these two values in Eq.(17) along with a rough estimate for $d(\overline{\Delta T_s}^{obs})/dt$ from Fig. 2, one obtains $H \approx 140 - 230$ meters. The order of magnitude of storage depth, H , values in this range is very reasonable (Ross, 1982).

Using the above (and $F_{out,o} \approx 250 W/m^2$, $T_{s,o} \approx 288^{\circ}K$, $H \approx 190$ meters, $\tau \approx \tau_*$) in Eq.(8a), one obtains:

$$\tau = \frac{0.13H}{4 - \phi_{fb}} \frac{T_{s,o}}{F_{out,o}} \approx \frac{28}{4 - \phi_{fb}} \text{ years}. \quad (18)$$

When one only knows the order of magnitude of τ , Eq.(18) can be used to provide a credible estimate of ϕ_{fb} . In contrast, Eq.(18) should *not* be used to estimate τ if ϕ_{fb} is expected to be an uncertain number close to 4.

7 Summary and concluding remarks

This paper has been updated using observational data published since 2008 (Morice, C. P. *et. al.*, 2012). This new data is completely consistent with that used previously as can be seen from Fig.1, 2.

- The data shown in Figs. 1 and 2 has been fit using least squares to yield values of the transient climate sensitivity ($\approx 2.3^\circ C$) and the thermal inertia time scale associated with global warming (≈ 32 years).

The single energy reservoir assumption is crucial to the derivation of Eq.(12) and Eq.(18) which are the main results of the paper. No detailed physics was required in the derivation of Eq.(12) which relates the Earth's equilibrium climate sensitivity $\lambda_{2X_o}^{eq}$ to its thermal inertial time scale.

- Eq.(12) also says that the lower limit of $\lambda_{2X_o}^{eq}$ is unlikely to be less than $2.3^\circ C$ since the factor τ/τ_T^{obs} must be positive. The IPCC best estimate of $\lambda_{2X_o}^{eq} \approx 3^\circ C$ would need $\tau \approx (3 - 2.3) \times (31/2.3) \approx 9.4$ years. A fat-tailed $\lambda_{2X_o}^{eq}$ beyond the IPCC $4.5^\circ C$ upper limit would need $\tau \geq 32$ years.
- Eq.(18) relates the thermal inertia time scale τ to the atmospheric feedback factor ϕ_{fb} and a parameter H which is an effective energy storage depth of the Earth's oceans. Using available observational data of the upper ocean heat content gives an estimated value of H between 140 and 230 meters.

Note that ΔT_s will continue to rise even after X/X_o has been successfully stabilized at some constant value. For example, if $\tau \approx 32$ years, then it would take more than six decades after X/X_o is stabilized in order for ΔT_s to eventually reach 90% of its full equilibrium rise. When τ is some multi-decadal number, the Earth's response to changes of radiative forcing is very sluggish. This sluggishness has important policy implications.

What is the probability that τ might be a multi-decadal or even multi-centurial number? It is not obvious that this question could

ever be answered *objectively*. The meaningfulness of probability distribution functions constructed by polling modeling data generated by computers is still subject to debate. At the present time, the available observational data do not support *subjective* assignments of multi-decadal τ 's.

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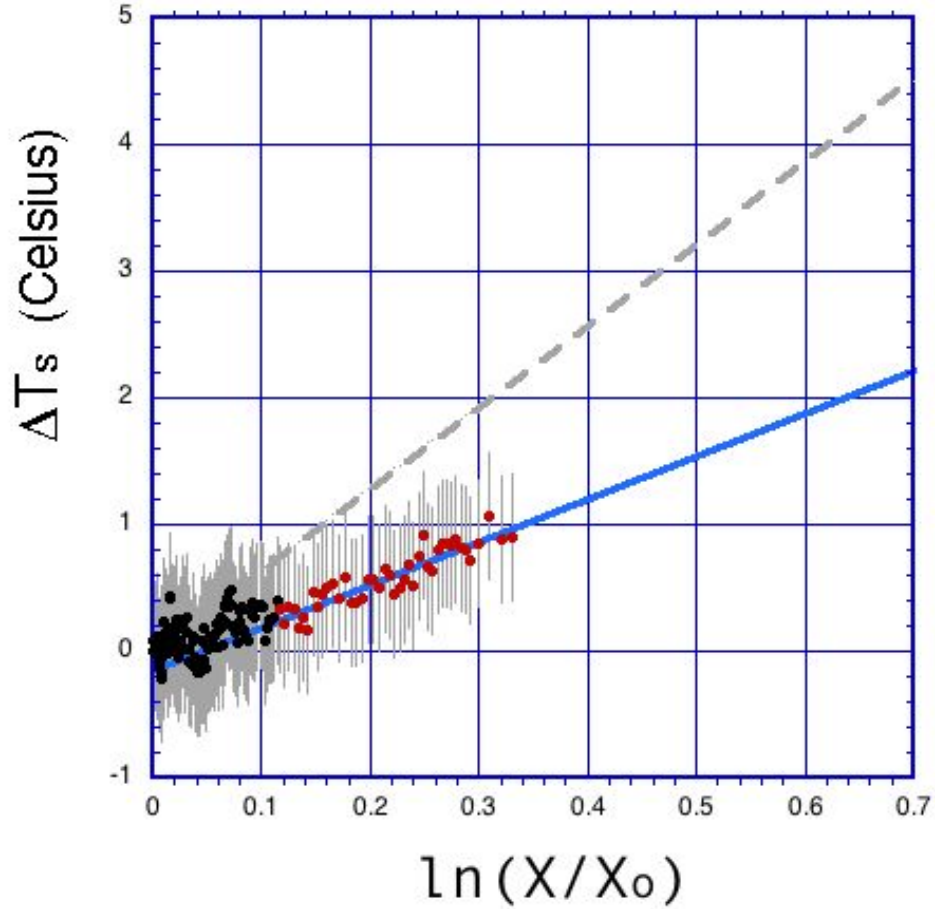


Figure 1: $\Delta T_s(^{\circ}C)$ vs $\ln(X/X_o)$ (1850-2012). Temperature data from Brohan, Kennedy, Harris *et. al.* (2006; HadCRUT3 dataset), Morice, Kennedy, Rayner *et. al.*, (2012) and CO_2 data from Enting, Wigley and Heimann (1994; see its Fig. B.1). Uncertainties in the individual $\Delta T_s(^{\circ}C)$ data are circa $\pm 0.5^{\circ}C$ (Frank, 2010). Solid line is the least squares best-fit straight line ($\lambda_{2X_o}^{obs} = 2.3^{\circ}C$; $R = 0.88$) for the post-1970 data ($\ln(X/X_o) > 0.13$). The dashed line is Eq.(1) plotted with $\lambda_{2X_o}^{eq} = 4.5^{\circ}C$ —which is the upper limit of the IPCC likely range. Note that $\ln 2 \approx 0.7$.

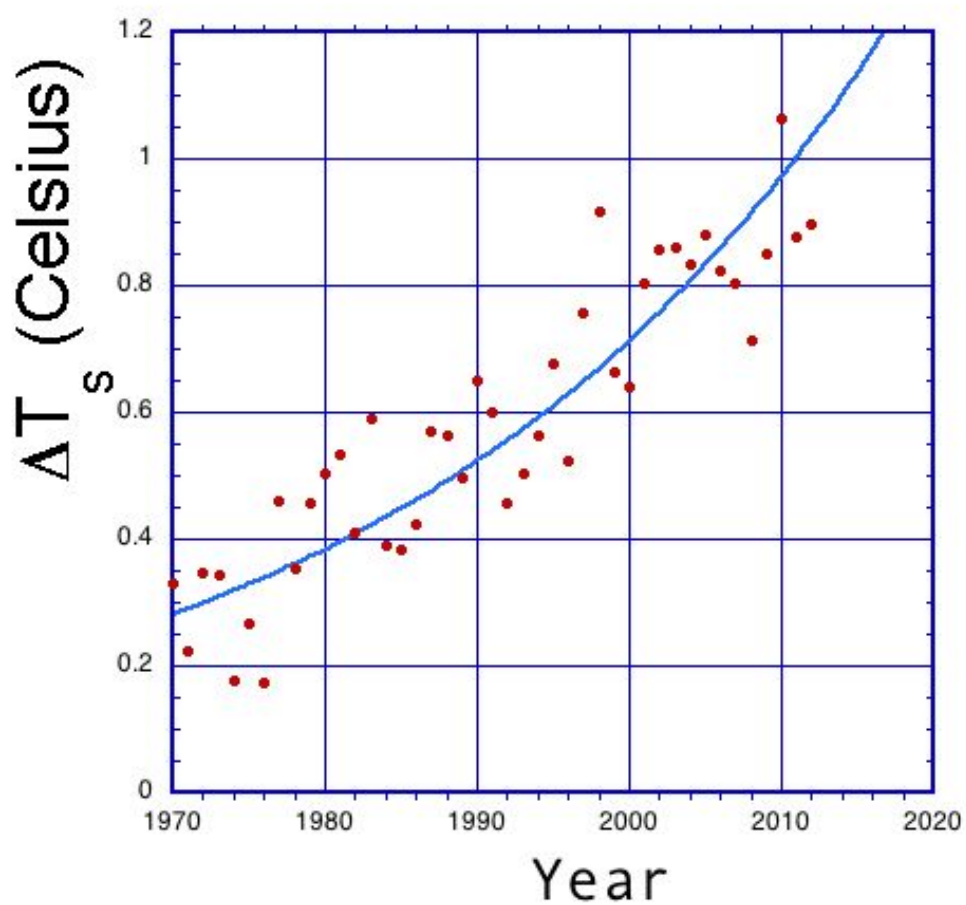


Figure 2: $\Delta T_s(^{\circ}C)$ vs time (Brohan *et al.*, 2006, Morice *et. al.*, 2012; post-1970 data only). Solid line is the least squares best-fit exponential line ($R=0.90$). The time scale of the exponential line is $\tau_T^{obs} \approx 32$ years. Uncertainties in the individual $\Delta T_s(^{\circ}C)$ data are circa $\pm 0.5^{\circ}C$ (Frank, 2010).